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Edge exponent in the dynamic spin structure factor of the Yang-Gaudin model

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The dynamic spin structure factor $\mathcal{S}(k,\omega)$ of a system of spin-1/2 bosons is investigated at arbitrary strength of the interparticle repulsion. As a function of ω it is shown to exhibit a power-law singularity at the threshold frequency defined by the energy of a magnon at given k. The power-law exponent is found exactly using a combination of the Bethe ansatz solution and an effective-field theory approach.

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The remarkable progress achieved by the theory of one-dimensional (1D) quantum fluids is rooted in the fact that dimensionality imposes severe constraints on the fluid's low-energy excitation spectrum. Due to these constraints the investigation of the low-energy dynamics of the fluid reduces to choosing the effective-field theory from a limited number of universality classes. Perhaps the most ubiquitous (and most thoroughly investigated) is the universality class called the Luttinger liquid.[1] Other nontrivial examples include states with non-Abelian currents and spin-incoherent[2–4] and ferromagnetic liquids.[5–9] For all such cases there exist well-developed analytical methods allowing one to calculate infrared asymptotics of dynamical correlation function, spectral properties, and scaling dimensions of local observables.

In a series of recent papers[5–19] it has been found that the dimensionality constraints and the resulting universality may extend far beyond the low-energy sector of the excitation spectrum of the fluid. It was shown that there exists a curve $\omega_{-}(k)$ in the (k,ω) space at which spectral functions exhibit power-law singularities of the type

$$\mathcal{S}(k,\omega) \simeq c(k)\theta[\omega-\omega_{-}(k)][\omega-\omega_{-}(k)]^{\Delta(k)}. \qquad (1)$$

Here, $\theta(x)$ is the Heaviside step function, $\Delta(k)$ and $c(k)$ are some momentum-dependent functions, and $\omega_{-}(k)$ is the energy of the lowest excited state of the fluid at a given momentum k. In a generic 1D fluid $\omega_{-}(k)>0$ for all k except a discrete set of points defined by the Luttinger theorem. The spectrum of momentum-dependent anomalous exponents $\Delta(k)$ in Eq. (1) is a natural generalization of the spectrum of scaling dimensions of the low-energy effective theory. Understanding the structure of the spectrum of $\Delta(k)$ will greatly advance the theory of 1D quantum fluids. There are several approaches to this problem. In Refs. 10 and 12 perturbation theory was used to get $\Delta(k)$ in a fermionic system. In Refs. 13–15 an effective-field theory approach establishing a link with the mobile quantum impurity problem[20–22] was proposed. This approach was complemented by Bethe ansatz (BA) calculations for several integrable models: Calogero-Sutherland,[11] Heisenberg,[16,17,19] and Lieb-Liniger.[18] Constraints on $\Delta(k)$ implied by symmetries of microscopic Hamiltonian were discussed in Refs. 8 and 15.

In a recent work[5] on the dynamical properties of a strongly repulsive ferromagnetic Bose gas, observable phenomena such as spin trapping and Gaussian damping of spin waves were predicted and a link between these phenomena and the singular behavior [Eq. (1)] of the dynamic spin structure factor was established. It was shown that, at infinite pointlike repulsion and for $k\to 0$,

$$\Delta(k) \simeq -1+\frac{K}{2}\left(\frac{k}{k_F}\right)^2, \qquad (2)$$

where K is the Luttinger parameter and $k_F=\pi\rho_0$ with ρ_0 being the average particle density. Assuming the validity of Eq. (2) at a large but finite repulsion, a crossover between trapped and open regimes of spin propagation was characterized completely. A different approach to the dynamics of the same system proposed in Ref. 6 confirmed Eq. (2). The approach of Ref. 6 was further developed in Ref. 7, demonstrating that for infinite pointlike repulsion $\Delta(k)$ has the form (2) for arbitrary k. In Ref. 8 the small k expansion of $\Delta(k)$ was shown to have the form (2) for arbitrary interparticle repulsion. However, the case of arbitrary k and interparticle repulsion remains unexplored.

In this Rapid Communication we investigate the behavior of $\Delta(k)$ and $\omega_{-}(k)$ for the dynamic spin structure factor of a ferromagnetic system of spin-1/2 bosons interacting through a pointlike repulsive potential of arbitrary strength. This system is described by the Yang-Gaudin model.[23] Combining the Bethe ansatz with an effective-field theory, we obtain our main result: explicit expressions (20)–(22) for $\Delta(k)$ at arbitrary k and interparticle repulsion.

The Hamiltonian of the Yang-Gaudin model is

$$H=\int_0^L dx[\partial_x\psi_\uparrow^\dagger\partial_x\psi_\uparrow+\partial_x\psi_\downarrow^\dagger\partial_x\psi_\downarrow+g\rho^2], \qquad (3)$$

where $\psi_{\uparrow(\downarrow)}(x),\psi_{\uparrow(\downarrow)}^\dagger(x)$ are canonical Bose fields satisfying periodic boundary conditions on a ring of circumference L and $\rho(x)$ is the total particle density operator. We consider the dynamic spin structure factor

$$\mathcal{S}(k,\omega)=\int dx\,dt\,e^{i(\omega t-kx)}\langle\Uparrow|s_+(x,t)s_-(0,0)|\Uparrow\rangle. \qquad (4)$$

Here, $s_+(x)=\psi_\uparrow^\dagger(x)\psi_\downarrow(x)$ is the local spin raising operator and $s_-(x)=[s_+(x)]^\dagger$. The average in Eq. (4) is taken with respect to a fully polarized ground state $|\Uparrow\rangle$ of the Hamiltonian (3) satisfying $s_+(x)|\Uparrow\rangle=0$ for all x. In the spectral representation Eq. (4) takes the form

$$\mathcal{S}(k,\omega)=\sum_f \delta(\omega-E_f(k))|\langle f,k|s_-(k)|\Uparrow\rangle|^2, \tag{5}$$

where the sum is taken over the eigenstates $|f,k\rangle$ of the Hamiltonian (3) carrying the momentum k. The energies $E_f(k)$ are defined by $H|f,k\rangle=E_f(k)|f,k\rangle$. The frequency $\omega_-(k)$ in Eq. (1) is given by $\omega_-(k)=\min_f E_f(k)$. Thus, the calculation of $\omega_-(k)$ reduces to the analysis of the energy spectrum of excitations. The calculation of $\Delta(k)$ directly from formula (5) is a far more difficult task. It requires the knowledge of the matrix element and their resummation procedure. For most integrable models, including Yang-Gaudin, such a calculation is beyond the reach of the existing theory. A way to bypass this problem is to combine the BA with an effective-field theory in a way similar to the derivation of scaling exponents in Luttinger liquids.[24–29] This is the route we take in our calculations.

We begin our analysis with a brief description of BA equations and a calculation of $\omega_-(k)$. A general solution to the Yang-Gaudin model is given by the nested BA.[23] All the states $|f,k\rangle$ in Eq. (5) lie in the sector with the z projection of the total spin given by $S_z=N/2-1$. In this sector Bethe's wave functions are characterized by a set of quasimomenta $\{\lambda_1,\ldots,\lambda_N,\xi\}$ satisfying

$$L\lambda_j+\sum_{k=1}^{N}\theta(\lambda_j-\lambda_k)=2\pi I_j+\theta(2\lambda_j-2\xi)+\pi. \tag{6}$$

Here, $\theta(\lambda)=2\arctan(\lambda/g)$ is the two-particle phase shift and $I_j=n_j-(N+1)/2$, where n_j are a set of distinct integers. The branch of $\theta(\lambda)$ is chosen so that $\theta(\pm\infty)=\pm\pi$. The total energy E and momentum P of a system are given by $E=\Sigma_{j=1}^N\lambda_j^2$ and $P=\Sigma_{j=1}^N\lambda_j$, respectively. The quasimomentum ξ enters in E and P indirectly, through the solution of Eq. (6). In the limit $\xi=\infty$ Bethe's equations (6) are identical to Bethe's equations of the fully polarized system,[30] $S_z=N/2$, which is equivalent to the Lieb-Liniger model.[31,32] The distribution of I_j in the ground state of the model is

$$I_j=j-\frac{N+1}{2},\quad j=1,\ldots,N. \tag{7}$$

Introducing the quasimomenta density $\rho(\lambda_j)=1/[L(\lambda_{j+1}-\lambda_j)]$ and taking the thermodynamic limit $0<\rho_0<\infty$ as $N,L\to\infty$, one gets the integral equation

$$\rho(\lambda)-\frac{1}{2\pi}\int_{-\Lambda}^{\Lambda}d\nu\,\rho(\nu)K(\lambda,\nu)=\frac{1}{2\pi} \tag{8}$$

for the quasimomenta in the state (7) and $\xi=\infty$. The kernel $K(\lambda,\nu)\equiv K(\lambda-\nu)$ is $K(\lambda)=\partial\theta(\lambda)/\partial\lambda=2g/(g^2+\lambda^2)$. Note that $\rho(\lambda)$ should satisfy $\int_{-\Lambda}^{\Lambda}d\lambda\,\rho(\lambda)=\rho_0$. This formula together with Eq. (8) is used to get the value of the Fermi quasimomentum Λ as a function of the particle density ρ_0. The ground-state energy in the thermodynamic limit is

$$E_0=L\int_{-\Lambda}^{\Lambda}d\lambda\lambda^2\rho(\lambda) \tag{9}$$

and the momentum of the ground state is zero.

Consider now the state characterized by a finite value of ξ and I_j given by their ground-state values [Eq. (7)]. This state is an excitation above the vacuum, which we shall call a magnon. Introducing the so-called shift function[33] by $F(\lambda_j|\xi)=(\lambda_j-\tilde{\lambda}_j)/(\lambda_{j+1}-\lambda_j)$, where λ_j are ground-state quasimomenta and $\tilde{\lambda}_j$ are those of the excited state, we get the following integral equation for F in the thermodynamic limit:

$$F(\lambda|\xi)-\frac{1}{2\pi}\int_{-\Lambda}^{\Lambda}d\nu\,K(\lambda,\nu)F(\nu|\xi)=-\frac{\pi+\theta(2\lambda-2\xi)}{2\pi}. \tag{10}$$

The momentum of the excited state is

$$k=\int_{-\Lambda}^{\Lambda}d\lambda\,\rho(\lambda)[\pi+\theta(2\lambda-2\xi)], \tag{11}$$

and its energy above the ground state is

$$\omega_-(k)=-\frac{1}{\pi}\int_{-\Lambda}^{\Lambda}d\lambda\,\varepsilon(\lambda)K(2\lambda-2\xi). \tag{12}$$

Here, ω_- is written as a function of the physical (observable) momentum k, which is related to the quasimomentum ξ by the integral equation (11). The quasienergy $\varepsilon(\lambda)$ is given by the solution of the integral equation

$$\varepsilon(\lambda)-\frac{1}{2\pi}\int_{-\Lambda}^{\Lambda}d\nu\,\varepsilon(\nu)K(\lambda,\nu)=\lambda^2-\mu \tag{13}$$

satisfying a condition $\varepsilon(\pm\Lambda)=0$. The parameter μ entering Eq. (13) is the chemical potential, defined by $\mu=(\partial E_0/\partial N)_L$, where E_0 is found from Eq. (9). One can show that at small k the dispersion law (12) is parabolic (Ref. 34), $\omega_-(k)=k^2/2m_*$, with the effective mass satisfying $m_*^{-1}=-(\pi g\rho_0^2)^{-1}\int_{-\Lambda}^{\Lambda}d\lambda\,\varepsilon(\lambda)$.

Another way to excite the system is to create a particle-hole pair by moving one of the quantum numbers I_j in Eq. (6) outside of the ground-state distribution (7). Such excitations are analyzed in detail in Ref. 32 (see also Ref. 33). In particular, at small momentum they are shown to be equivalent to sound waves propagating at the velocity

$$v_s=\frac{1}{2\pi\rho(\Lambda)}\left.\frac{\partial\varepsilon(\lambda)}{\partial\lambda}\right|_{\lambda=\Lambda}. \tag{14}$$

Any excitation in the N-particle sector with $S_z=N/2-1$ consists of several particle-hole pairs and one magnon. The exact lower bound of the particle-hole continuum[32] and the dispersion curve of the magnon are illustrated in Figs. 1(a) and 1(b). For all values of the coupling constant g the magnon branch lies below the particle-hole continuum. It is thus the single magnon dispersion [Eq. (12)] that gives the exact lower bound of the excitation spectrum.

While $\omega_-(k)$ is found using BA exclusively, in order to get

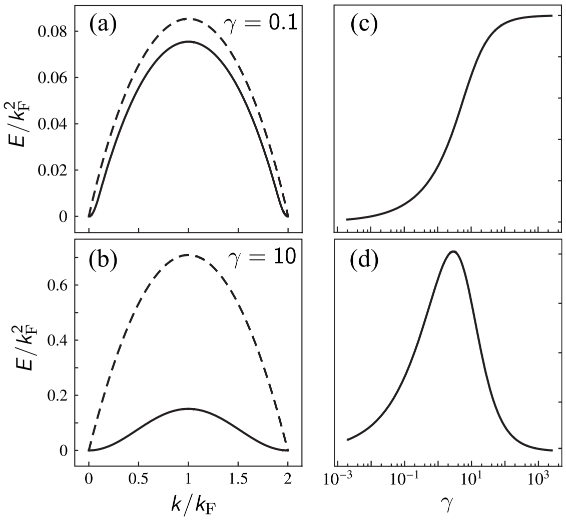

FIG. 1. Excitation spectrum in the model (3). (a) and (b) show the dispersion curve $\omega_-(k)$ [Eq. (12)] of the magnon (solid line) and the exact lower bound of the particle-hole continuum (dashed line) for two values of $\gamma=g/\rho_0$. (c) shows the γ-dependent exact lower bound E_p of the particle-hole continuum at $k=k_F\equiv\pi\rho_0$. (d) shows the γ-dependent magnon energy $E_m=\omega_-$ at $k=k_F$.

the threshold exponent $\Delta(k)$ in Eq. (1) for the function (4) we need to combine the BA solution with a low-energy effective-field theory. To do so, we adapt the method proposed in Ref. 17 to the nested BA case discussed here. By doing so we reduce the initial problem to the problem of a Luttinger liquid minimally coupled to the infinitely heavy spin degree of freedom. The latter is solved explicitly by a unitary transformation. As a first step we introduce an auxiliary microscopic theory with a local Hamiltonian $\widetilde{H}$ depending on k as an external parameter and having the following properties: (i) it conserves the total momentum, which will be denoted by q; (ii) its excitation spectrum at $q=k$ is gapless; and (iii) its structure factor $\widetilde{\mathcal{S}}$ satisfies

$$\frac{\widetilde{\mathcal{S}}(q,\omega)}{\mathcal{S}(q,\omega_-(k)+\omega)}\to 1, \quad q=k, \quad \omega\to 0. \tag{15}$$

In integrable models $\widetilde{H}$ can be constructed as a linear combination of a finite number of mutually commuting local integrals of motion. The eigenstates $|f,q\rangle$ of H are at the same time the eigenstates of $\widetilde{H}$; therefore, $\widetilde{\mathcal{S}}(q,\omega)=\Sigma_f\delta(\omega-\widetilde{E}_f(q))|\langle f,q|s_-(q)|\Uparrow\rangle|^2$, where $\widetilde{H}|f,q\rangle=\widetilde{E}_f(q)|f,q\rangle$. Like for H, the low-energy spectrum of $\widetilde{H}$ consists of sound waves and a magnon. The energy of the magnon is proportional to $(k-q)^2$ as $q\to k$. Condition (15) requires that the velocities of the right- and left-moving sound waves be different and given by[35]

$$v_\pm = v_s \pm \partial\omega_-(k)/\partial k, \tag{16}$$

where v_s is given by Eq. (14).

The dynamics of sound waves is governed by the Luttinger Hamiltonian

$$H_0=\sum_{r=\pm}H_r, \quad H_r=\frac{v_r}{4\pi}\int_0^L dx{:}[\partial_x\varphi_r(x)]^2{:}, \tag{17}$$

where the operators φ_r are chiral boson fields $[\varphi_r(x),\varphi_{r'}(x')]=i\pi r\delta_{rr'}\,\mathrm{sgn}(x-x')$ related to the microscopic particle density by

$$\rho(x)=\rho_0+(2\pi)^{-1}\sqrt{K}[\partial_x\varphi_+(x)-\partial_x\varphi_-(x)] \tag{18}$$

and the symbol :: stands for the boson normal ordering. In order to describe the low-energy magnon excitation we introduce the spin-density field $\tilde{\mathbf{s}}(x)$, related to the microscopic spin density by $s_z(x)=\tilde{s}_z(x)+\rho_0/2$ and $s_\pm(x)=e^{\pm ikx}\tilde{s}_\pm(x)$, where $s_\pm=s_x\pm is_y$ are the local spin-ladder operators of Eq. (4). Within the effective theory the operators $\tilde{s}_\pm$ are smooth spin-flip fields. Generally, a local spin flip may excite sound waves. Thus, an effective theory should contain a coupling between $\tilde{\mathbf{s}}$ and $\varphi_\pm$. There is no general prescription on how to derive the corresponding coupling term microscopically. Here, we construct it as the minimal local coupling respecting the SU(2) symmetry of the microscopic theory, and vanishing in the absence of magnon excitations[36]

$$H_i=-\sum_{r=\pm}\frac{v_r\beta_r}{2\pi}\int_0^L dx\;\partial_x\varphi_r(x)\tilde{s}_z(x). \tag{19}$$

Other possible couplings involve higher gradient terms and higher harmonics of the density operator (18). Those are infrared irrelevant and do not contribute to the critical exponents. The kinetic-energy density of the spin field is represented by a higher gradient term $\partial_x\tilde{s}_+(x)\partial_x\tilde{s}_-(x)$ that can also be neglected in the calculation of the critical exponents.[37] The total Hamiltonian of the effective theory describing the dynamics near the threshold is thus given by $H_{\mathrm{eff}}=H_0+H_i$. This Hamiltonian is diagonalized by a unitary transformation $e^{iS}H_{\mathrm{eff}}e^{-iS}$ with $S=(2\pi)^{-1}\int_0^L dx[\beta_+\varphi_+(x)-\beta_-\varphi_-(x)]\tilde{s}_z(x)$. For the function (4) this gives

$$\Delta(k)=-1+\frac{1}{4\pi^2}(\beta_+^2+\beta_-^2). \tag{20}$$

What remains is to determine the coupling constants $\beta_\pm$ in terms of the parameters of the microscopic theory. This is done by the comparison of the low-energy spectrum of the microscopic Hamiltonian $\widetilde{H}$, found from the BA solution, and the spectrum of the effective Hamiltonian H_{eff}.[33] This procedure yields

$$\beta_r=2\pi rF(r\Lambda|\xi), \quad r=\pm 1, \tag{21}$$

where F is defined by the solution of the integral equation (10). We solve this equation and find $\Delta(k)$ numerically for different values of the coupling constant $\gamma=g/\rho_0$. For easier comparison with Eq. (2) we represent our result in the form

$$\Delta(k)=-1+\frac{K}{2}\left(\frac{k}{k_F}\right)^2+\frac{(K-1)^2}{K}\alpha(k), \tag{22}$$

where $k_F=\pi\rho_0$ and $K=k_F/v_s$ is the Luttinger parameter calculated using Eq. (14).

The function $\alpha(k)$ for different values of the Luttinger parameter is shown in Fig. 2. One can see that at large values

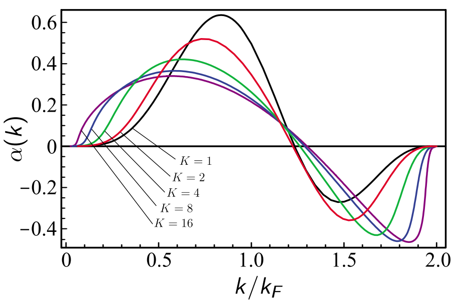

FIG. 2. (Color online) The function $\alpha(k)$ on the right hand side of Eq. (22) is plotted for different values of the dimensionless coupling constant γ. The values of the Luttinger parameter K are indicated for each curve and correspond in increasing order to $\gamma=\infty$, 1.65, 0.56, 0.238 and 0.109, respectively.

of γ (or K close to 1) the small k expansion derived in Ref. 5 is valid for all values of k, in agreement with Ref. 7. It is interesting to note that the leading correction to this result is of the second order, $(K-1)^2 \sim \gamma^{-2}$. At arbitrary interaction strength $\alpha(k) \sim k^4$ as $k \to 0$; therefore, the small k expansion of $\Delta(k)$ found in Ref. 5 remains valid for all values of γ, confirming the general result of Ref. 8. Note that $\alpha(k)$ also vanishes at $k=2k_F$.

The problem considered in the present work is directly related to the x-ray edge problem in the theory of the mobile impurity. In this context, the model (3) was investigated in Ref. 22. The approach of Ref. 22 exploits a transformation to the comoving reference frame and combines BA with an effective-field theory similar to ours. The method of Ref. 22 has recently been successfully applied to the Heisenberg model and later was shown[19] to produce results equivalent to the method of Ref. 17 used here. A direct comparison of the present work with Ref. 22 is however not possible, because the latter is largely based on a Bethe ansatz solution[38] violating continuity conditions.

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